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Investigation of Heat Transfer in Pipes Using Dimensionless Numbers

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Abstract

This study investigates heat transfer in pipes, focusing on the calculation of three key dimensionless numbers to analyze thermal behavior. Although the experiment was conducted only once, measures were taken to ensure accuracy, and potential sources of error were minimized. Repeating the experiment multiple times and averaging the results would have further improved reliability by reducing the impact of anomalies. Despite this limitation, the primary objective of understanding heat transfer mechanisms through dimensionless numbers was successfully achieved, providing valuable insights into thermal processes.

Key Words:

Dimensionless numbers, thermal behavior, accuracy, anomalies

Introduction

This article focuses on heat transfers to the fluid flowing inside a pipe. It is assumed from here on that we assume fully developed incompressible, Newtonian, steady flow conditions. Fully developed flow implies that the tube is long compared with the entrance length in which the velocity distribution at the inlet adjusts itself to the geometry and no longer changes with the distance along the tube [1].

Background theory

Heat Exchangers are widely used in a variety of applications and heat transfer occurs between the inner surface of the pipe and the fluid via convection. In order to analyze the heat exchanger, the heat transfer co-



efficient between the wall of the conduit and the fluid flowing inside it is required [2]. This can be found from the Nusselt number equation,

$$Nu = \frac{\alpha L}{K}$$



Where; α = Heat transfer coefficient, L = Characteristic Length and K = Thermal Conductivity

After the coefficient α is obtained, the heat transfer rate q can also be calculated by the equation given below;

$$q = (T_1 - T_2)$$

It is clear to see from the equation, that there are several physical variables which affect the heat transfer process. These variables are the surface area (A), the surface temperature of the pipe (T_1) and the temperature of the fluid flowing through the pipe (T_2) [3]. In terms of the heat transfer co-efficient it is important to note that it is principally dependent on two dimensionless parameters, the Reynolds number, (Re) and the Prandtl number, (Pr). The Reynolds number is used to define whether to flow is laminar or turbulent. The importance of this is further highlighted based on the fact that the heat transfer rate is higher for turbulent flow than for laminar flow [4]. The equation for the Reynolds number is as follows,

$$Re = \frac{\rho u D_H}{\mu}$$

Where ρ = density of the fluid, u = mean velocity, D_H = pipe diameter, and μ = dynamic viscosity.

The Prandtl number is the ratio between kinematic viscosity and the thermal diffusivity, however unlike the Reynolds number; it does not contain a length scale variable, and is solely dependent on the fluid itself and its state. The heat transfer in fluids with a high Prandtl number rely more on the turbulence of the flow as oppose to the diffusivity from the wall [5]. It is also important to note that the Prandtl number controls the relative thickness of the momentum and thermal boundary layers, and so when the number is small it means the heat diffuses very quickly compared to the velocity (momentum) [6]. The Prandtl number can be found from the following equation,

$$Pr = \frac{C_p \mu}{k}$$

; Where C_p = specific heat, μ = dynamic viscosity and k = thermal conductivity. Stanton number, (S) is a dimensionless number used mainly on forced convection. It is defined as a dimensionless number that measures the ratio of heat transferred into a fluid to the thermal capacity of that fluid. From this it is clear that the Stanton number is a function of the Nusselt number, Reynolds number and also the Prandtl number [7]. It is given by the following equation,

$$\alpha$$



$$St =$$

$$C_p \rho U$$

Where α = heat transfer co-efficient, C_p = specific heat, ρ = density of the fluid, and U = velocity of the fluid.

Apparatus and procedure

The apparatus consists of a steel pipe fitted with five heaters attached with equal distances. These heaters are thermostatically controlled to maintain a constant temperature of 80°C. A venture meter is also attached to help out with the calculations of the flow rate of air. A water manometer records the difference in pressure in the venture [8]. A fan is attached at the end of the pipe which sucks in air through and the outlet of the air is controlled by four orifices of diameter 20mm, 27mm, 40mm and 65mm respectively. Atmospheric pressure and room temperature was recorded before the experiment. The heaters were switched on prior to the experiment and thus the wall temperature for the heated part was uniform. The wall temperatures were also recorded [9]. The first orifice that was attached had a diameter of 20mm. The manometer reading for inlet-throat and inlet-atmosphere was recorded. By means of selector switch, power dissipated in each of the five heaters were also noted down. Each heater used had a total rated power of 1200W but for this experiment the percentage power dissipated for each heater was recorded. In order to achieve a different flow rate, the orifice on the fan exit was changed. These orifices had different diameter as discussed above. After every change of the orifice, 15-20 minutes were given to the system for it to reach steady conditions. Table 1.1 shows the readings recorded



TABLE 1.1

Orifice Diameter er (mm)	Pressure(mmH2O)		Air Temperatu re (°C)		Heaters									
					Inlet 1		2		3		4		5	
Inlet- throat	Inlet- Atmosphe re	Inlet	Outle t	T/ °C	%P	T/ °C	%P	T/ °C	%P	T/ °C	% P	T/ °C	%P	
20	24+34	8+16	20	44	80	14	80	9	80	9	80	9	80	8
27	82+70	33+26	20	42	80	19	80	11	80	11	80	12	80	10
40	157+140	56+52	20	40	80	23	80	12	80	15	80	15	80	15
65	200+180	70+65	20	39	80	24	80	13	80	16	80	16	80	16

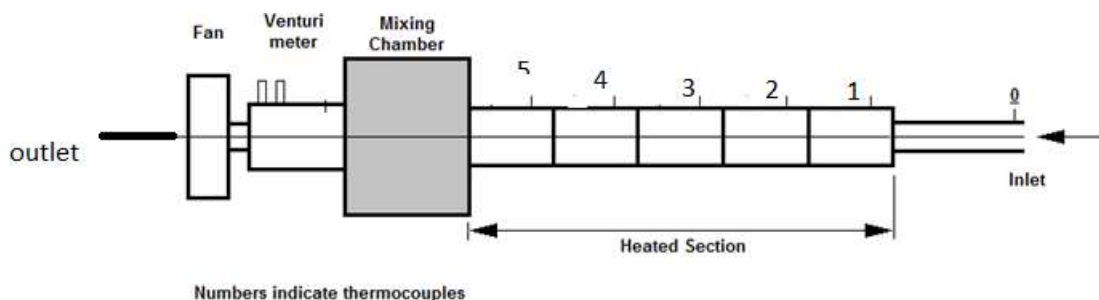


Diagram 1.1

Diagram 1.1 represents the apparatus set up in the laboratory and is based on the discussion written above.

Calculations

Calculations shown in this sub-heading is for the 20mm diameter orifice. For all the other orifices, the calculations follow the same pattern. During the experiment, the room temperature recorded was 22.5°C and the atmospheric pressure was 782mmHg. The calculations shown below have values taken from Table 1.1.

Air density is calculated using the equation;

$$\rho = \frac{P}{RT} \quad \text{(Equation 1.1)}$$

Where; ρ = Air density in kg/m^3 , P = Absolute Pressure in Pa, R = Specific Ideal Gas Constant and T = Temperature at inlet to the venture meter

104258.09

$$\rho = \frac{104258.09}{287.10 \times 295.65}$$

$$\rho = 1.228 \text{ kg/m}^3$$

Mass flow rate of the air is obtained by;

$$m = K\sqrt{h} \quad \text{(Equation 1.2)}$$

Where; $K = 1.689^m \sqrt{g}$, h = manometer reading converted to Pa and ρ = Air density



$$m = 1.689 \times \sqrt{(1.228) \times (0.024 + 0.034)}$$

$$m = 0.4508 \text{ Kg/s}$$

Total heat-transfer rate \dot{Q}_{tot} to the air is calculated using the equation;

$$\dot{Q}_{tot} = (\theta_{outlet} - \theta_{inlet}) \text{ (Equation 1.3)}$$



Where; c_p = isobaric specific heat capacity of air taken constant at 1005 J/kgK

$$\dot{Q}_{tot} = 0.4508 \times 1005 \times (317.15 - 293.15)$$

$$\dot{Q}_{tot} = 10873 \text{ J/s}$$

Total power dissipated in all five heaters;

$$P = 168 + 108 + 108 + 108 + 96$$

$$P = 588 \text{ W}$$

Hence, the heat transfer rate to the surrounding;

$$\dot{Q}_s = P - \dot{Q} \quad (\text{Equation 1.4})$$

$$\dot{Q}_s = 588 - 10873$$

$$\dot{Q}_s = -10285 \text{ J/s}$$

Heat transfer rate in the first section;

$$\begin{aligned} \dot{Q}_1 &= \dot{Q}_s \quad (\text{Equation 1.5}) \\ &= P - \\ &5 \end{aligned}$$

$$\dot{Q}_1 = 2645 \text{ J/s Heat}$$

transfer rate to the air in other four sections;

$$\dot{Q}_{2-5} = \dot{Q}_{tot} - \dot{Q}_1$$

$$\dot{Q}_{2-5} = 10873 - 2645 \text{ J/s}$$

$$\dot{Q}_{2-5} = 8228 \text{ J/s}$$

Mean air temperature at the end of section 1;

$$\theta^* = \frac{\dot{Q}_1}{mc_p} + \theta \quad (\text{Equation 1.6})$$

$$mc_p \quad \text{inlet}$$

$$\theta^* = \frac{2645}{0.450}$$



$$8 \times 1005 \quad + 293.15$$

$$\theta^* = 299.0 \text{ K}$$

To calculate surface heat transfer coefficient, the log mean temperature difference is calculated for;

- a) The total heated part i.e. for all the five sections
- b) The first section only
- c) Sections 2 and 5 together

Log-mean temperature is calculated using the formula;



$$\Delta T_m = \frac{\theta_{out} - \theta_{in}}{\ln \left[\frac{\theta_w - \theta_{in}}{\theta_w - \theta_{out}} \right]} \quad (\text{Equation 1.7})$$

Hence, the log mean temperature for **a)** is calculated as shown below;

$$\Delta T_m = \frac{317.15 - 293.15}{\ln \left[\frac{353.15 - 293.15}{353.15 - 317.15} \right]}$$

$$\Delta T_m = 46.98 \text{ K}$$

Heat transfer surface area for **a)**;

$$A = \quad (\text{Equation 1.8})$$

$$A = 3.14 \times 0.0379 \times 1.5$$

$$A = 0.179 \text{ m}^2$$

Heat transfer surface area for **b)**;

$$A = \pi dl$$

$$A = 3.14 \times 0.0379 \times 0.3$$

$$A = 0.0357 \text{ m}^2$$

Heat transfer surface area for **c)**;

$$A = \pi dl$$

$$A = 3.14 \times 0.0379 \times 1.2$$

$$A = 0.1428 \text{ m}^2$$

Surface heat transfer coefficient for **a)**;

$$\alpha$$

$$= \alpha =$$



$$\alpha = \frac{Q_{to}}{t \Delta T_m} = \frac{10873}{0.179 \times 46.98} = 1293 \frac{\text{J}}{\text{m}^2\text{sK}}$$

Log mean temperature difference for **b**);

$$\Delta T_m = \frac{\theta_{out} - \theta_{in}}{\ln \left[\frac{\theta_w - \theta_{in}}{\theta_w - \theta_{out}} \right]} = \frac{308.35 - 293.15}{\ln \left[\frac{353.15 - 293.15}{353.15 - 308.35} \right]}$$



$$\Delta T_m = 52.03 \text{ K}$$

Surface heat transfer coefficient for b);

$$\dot{Q}_1$$

$$\alpha = \frac{\dot{Q}_1}{A \Delta T_m}$$

$$2645$$

$$2645$$

$$\alpha =$$

$$\frac{0.0357 \times 52.03}{\text{m}^2 \text{sK}}$$

$$\alpha = 1424 \frac{\text{J}}{\text{m}^2 \text{sK}}$$

Log mean temperature difference for c);

$$\theta_{out} - \theta_{in}$$

$$\Delta T_m =$$

$$\ln \left[\frac{\theta_w - \theta_{in}}{\theta_w - \theta_{out}} \right]$$

$$\theta_w - \theta_{out}$$

$$317.15 - 308.35$$

$$\Delta T_m =$$

$$\frac{353.15 - 308.35}{\ln \left[\frac{353.15 - 317.15}{353.15 - 308.35} \right]}$$

$$\ln [$$

$$]$$

$$353.15 - 317.15$$

$$\Delta T_m = 40.24 \text{ K}$$

Surface heat transfer coefficient for c);

$$\dot{Q}_{2-5}$$

$$\alpha = \frac{\dot{Q}_{2-5}}{A \Delta T_m}$$

$$8228$$

$$8228$$

$$\alpha =$$

$$\frac{0.1428 \times 40.24}{\text{m}^2 \text{sK}}$$



$$\alpha = 1432 \frac{\text{J}}{\text{m}^2\text{sK}}$$

Stanton number for **a**);

$$St = \frac{\alpha \pi d^2}{4mc_p}$$

$$St = \frac{1293 \times \pi \times (0.0379)^2}{4 \times 0.4508 \times 1005}$$
$$St = 3.220 \times 10^{-3}$$

Stanton number for **b**);

$$St = \frac{\alpha \pi d^2}{4mc_p}$$
$$St = \frac{1424 \times \pi \times (0.0379)^2}{4 \times 0.4508 \times 1005}$$



$$St = 3.546 \times 10^{-3}$$

Stanton number for c);

$$St = \frac{\alpha \pi d^2}{4 m c_p}$$
$$St = \frac{1432 \times \pi \times (0.0379)^2}{4 \times 0.4508 \times 1005}$$
$$St = 3.566 \times 10^{-3}$$

Reynolds number for a);

$$Re = \frac{4m}{\pi d \mu}$$
$$Re = \frac{4 \times 0.4508}{\pi \times 0.0379 \times 0.00045}$$

Reynolds number for b);

Prandtl Number for a);

Reynolds number for c);



$$Re = \frac{\pi \times 0.0379 \times (18.7 \times 10^{-6})}{4m}$$

$$Re = 809866$$

$$Re = \frac{\pi \times 0.0379 \times (19 \times 10^{-6})}{4m}$$

$$Re = 797078$$

$$Re = \frac{4m}{\pi d \mu}$$

$$Re = \frac{4 \times 0.4508}{\pi \times 0.0379 \times (18.5 \times 10^{-6})}$$

$$Re = 818621$$

$$Pr = \frac{\mu C_p}{k}$$

$$Pr = \frac{26.5}{(18.7 \times 10^{-6} \times 1005)}$$

$$Pr = 7.09 \times 10^{-4}$$

Prandtl Number for **b**);



$$\text{Pr} = \frac{\mu C_p}{k}$$

$$\text{Pr} = 7.12 \times 10^{-4}$$

Prandtl Number for c);

$$\text{Pr} = \frac{\mu C_p}{k}$$

$$\text{Pr} = 7.07 \times 10^{-4}$$

Representation of the result

For this part of the section, the results are organized in a table and drawn into a graph for better representation and understanding. Discussion will then follow from after the pattern of the results achieved has been made clear under this heading.

Table 1.2, 1.3, 1.4 and 1.5 below represents the Stanton Number, Reynolds Number and Prandtl Number for orifices of diameter 20mm, 27mm, 40mm and 65mm respectively.

TABLE 1.2

---	Stanton Number	Reynolds Number	Prandtl Number
Total Heated Part	0.00322	809866	0.000709
First Section Only	0.00355	797078	0.000712
Section 2-5 Only	0.00357	818621	0.000707



TABLE 1.3

---	Stanton Number	Reynolds Number	Prandtl Number
Total Heated Part	0.00288	1317960	0.000711
First Section Only	0.00315	1317960	0.000716
Section 2-5 Only	0.00331	1280107	0.000712

TABLE 1.4

---	Stanton Number	Reynolds Number	Prandtl Number
Total Heated Part	0.00255	1809780	0.000711
First Section Only	0.00248	1870106	0.000704
Section 2-5 Only	0.00326	1790527	0.000713

TABLE 1.5

---	Stanton Number	Reynolds Number	Prandtl Number
Total Heated Part	0.0024	2084317	0.000711
First Section Only	0.00229	2153794	0.000709
Section 2-5 Only	0.00292	2062143	0.000716



Table 1.6, 1.7, 1.8 and 1.9 below represents $\ln(St)$, $\ln(Re)$ and Prandtl Number for orifices of diameter 20mm, 27mm, 40mm and 65mm.

TABLE 1.6

---	$\ln(St)$	$\ln(Rt)$	Prandtl Number
Total Heated Part	-5.738	13.604	0.000709
First Section Only	-5.641	13.589	0.000712
Section 2-5 Only	-5.635	13.615	0.000707

TABLE 1.7

---	$\ln(St)$	$\ln(Rt)$	Prandtl Number
Total Heated Part	-5.849	14.092	0.000711
First Section Only	-5.76	14.092	0.000716
Section 2-5 Only	-5.711	14.062	0.000712

TABLE 1.8

---	$\ln(St)$	$\ln(Rt)$	Prandtl Number
Total Heated Part	-5.972	14.409	0.000711
First Section Only	-6	14.442	0.000704
Section 2-5 Only	-5.726	14.398	0.000713

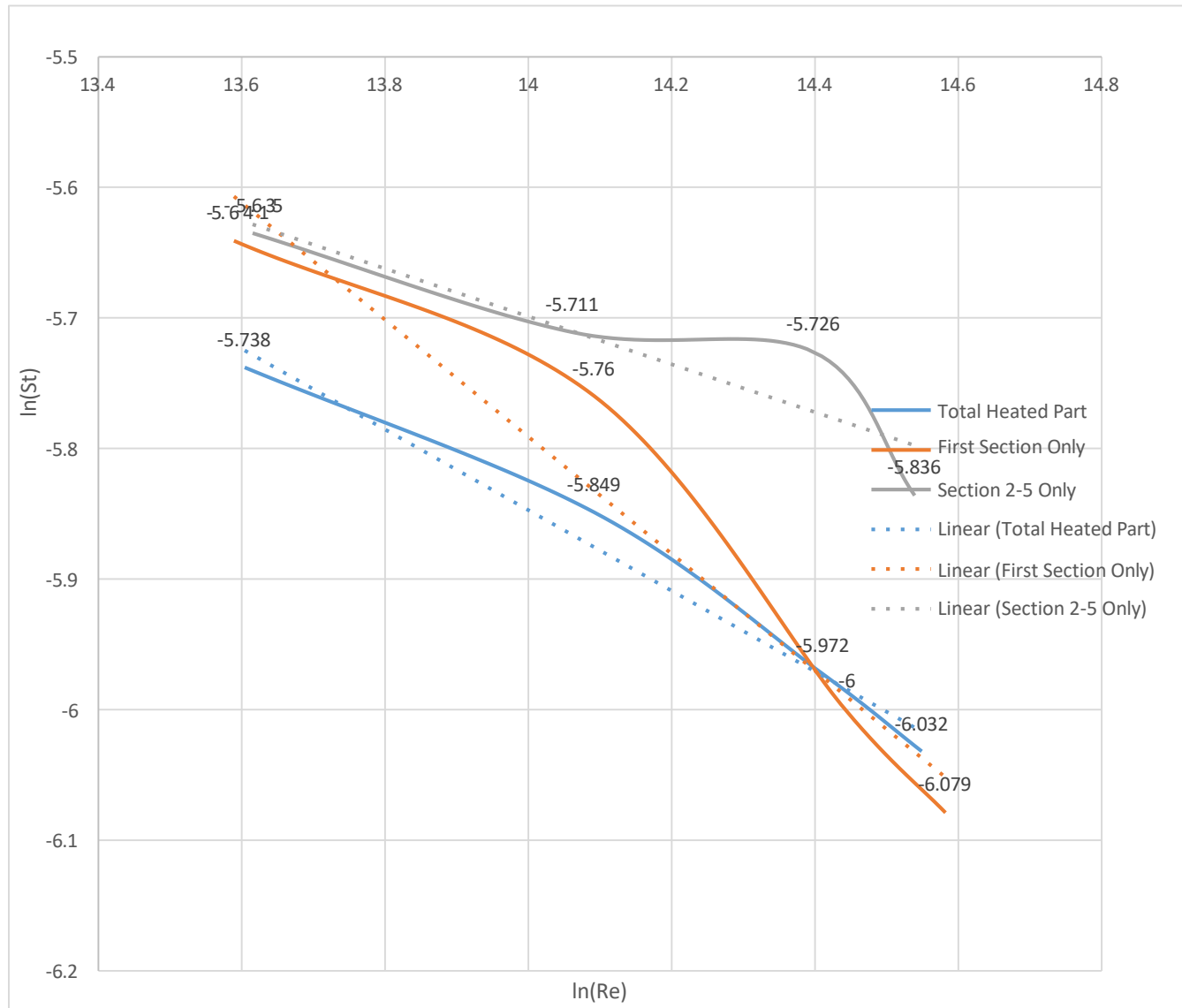


TABLE 1.9

---	$\ln(St)$	$\ln(Rt)$	Prandtl Number
Total Heated Part	-6.032	14.549	0.000711
First Section Only	-6.079	14.582	0.000709
Section 2-5 Only	-5.836	14.539	0.000716



A graph of $\ln(St)$ vs $\ln(Re)$ is drawn below.



Discussion

The Reynolds number, (Re) is sufficiently high for all of the flow rates and to be precise [10-13]. If Reynolds Number is under 2300, the flow is laminar. If its Re is between 2300 and 4000 the flow is transient and if it's more than 4000 the flow is turbulent. For the results in discussion in this particular article, the Reynolds Number has come up well above 4000. Thus the flow is turbulent and Dittus Boelter correlation is valid. Turbulent flow is a mixture of hot and cold fluid, which leads to higher heat transfer coefficients in comparison to the laminar region. Furthermore it is important to note that between these two regions therein lies a transitional region where a small amount of mixing occurs; however this shouldn't be looked into excessively as heat transfer by conduction is still imperative. Prandtl number remained relatively constant throughout the experiment. There has been very little variation which is very insignificant and thus holds none or very little importance in the overall results. Furthermore this small variation could have been a consequence of the amount of significant figures used during calculation, and thus should be neglected.

Another key aspect of this experiment which needs to be mentioned is the entrance effects. Because of no-slip condition, the fluid particles in the layer in contact with the surrounding of the pipe come to a complete stop. This layer also causes the fluid particles in the adjacent layers to slow down gradually as a result of friction. Also, as a fluid enters a pipe with uniform velocity, the fluid immediately adjacent to the wall is brought to rest. A laminar boundary layer forms at the wall for a short distance along the pipe, however if the fluid entering has a high turbulence, the boundary layer quickly becomes turbulent also. The results of this experiment show that the entrance effect has great significance. It is important to note that the development of the thermal boundary layer is qualitatively the same as that of the hydrodynamic boundary layer, and as the temperature develops with the increasing distance, the heat transfer co-efficient decreases.

In this experiment, one of the focal aspects was the accuracy of the instruments used and several possible sources of errors. This was essential in calculating the Reynolds Number, Stanton Number and Prandtl Number, as any minor mishaps could cause severe variations within the results. The thermocouple and wattmeter had continuous variation in the output value they showed that might have also contributed to the variation of results. Wattmeter has an accuracy of $\pm 5\%$. Therefore, there are uncertainties in the value of the wattmeter which may have carried forward to the data and results done in this article. The Thermocouple in particular took time to stabilize, which meant if results were recorded too early or before it had properly settled, there would be a strong possibility that these results were incorrect. In terms of the venturi meter, there was also a possible source of parallax errors.

Furthermore the mercury in the venturi meter had a curvature (meniscus) which was not taken into consideration when recording the results,

and therefore the results were subjective. According to the standard, the top of the meniscus was read. The inaccuracy of these initial readings may have altered the calculations done in this article. Small errors in the beginning results to greater degree of error at the end when the initial errors are carry forward.

The results from this experiment tell us that the correlation is best suited for smooth pipes, so any commercial applications whereby rough tubes are applied should be avoided. Additionally when there is a large temperature difference within the fluid being analyzed, the correlation automatically becomes less accurate.

APPENDIX

TABLE 2.0

Orifice Diameter(mm)	Mass flow- rate(kg/s)	Q(total) (W)	Q1 (W)	Q(2- 5)(W)	Avg Stanton Number	Avg Reynolds Number	Avg Prandtl Number
20	0.4508	10873	2645	8228	0.00344	808522	0.000709
27	0.7297	16134	3697	12437	0.00311	1305342	0.000713
40	1.02	20502	4571	15931	0.00277	1823471	0.000709
65	1.154	22031	4877	17154	0.00253	2100085	0.000712

Conclusion

There are several sources of error which may have come in to this article. Every action has been taken to make results as accurate as possible. Results would have been more accurate of the experiment would have been conducted twice or thrice. Averages could then have been calculated to minimize any further errors. This would have reduced the chances of using anomalous results and hence increase the reliability of the final results. Regardless of this, the primary aim of investigating the heat transfer in pipe was fulfilled, and the results for the three dimensionless numbers were obtained successfully.

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